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Numerical analysis of plasma instabilities in the TEXTOR tokamak edge plasma

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Abstract

In the present paper we study the evolution of Kelvin–Helmholtz and short-wavelength interchange instabilities in the TEXTOR tokamak boundary layer. The unstable modes are related to the drift flows and develop in regions of steep radial gradients close to the separatrix and in regions of unfavorable curvature, especially in the vicinity of the limiter. We investigate the behavior of these instabilities as a function of the numerical grid spacing and for different plasma parameters in the SOL. We have found that the most dangerous one is the Kelvin–Helmholtz instability, which arises at the ion side of the limiter. If a special kind of radial damping of the fluctuations is introduced, then it is possible to kill the KH instability and consequently the interchange instability can show up. We observed also short-wave oscillations which are localized in the transition layer.

Keywords: Plasma modeling; SOL modeling; 2D fluid code; Drifts; Edge transport

1. Introduction

Scrape-off layer (SOL) turbulence is observed in most fusion discharges and produces anomalous radial transport of particles and energy in the boundary plasma [1,2]. Its obvious signatures are large amplitude fluctuations of plasma density, temperature, and plasma potential. The study of these fluctuations is important, as they play a central role in determining the scrape-off width, and thus the magnitude of the peak heat load on the divertor or limiter plates.

In this paper we present analytical and numerical results related to the nonlinear evolution of the Kelvin–Helmholtz and interchange instabilities in the boundary layer of the TEXTOR tokamak. Investigations have been carried out by means of 2D numerical simulations with the multi-fluid code TECXY [3]. The physical model is based on Braginskij-like equations for the background plasma and rate equations for the impurity ions. The present version of the code incorporates drift motions and currents in a fully self-consistent way with plasma and impurity dynamics in the curvilinear geometry of the TEXTOR limiter tokamak boundary layer.

It appears that the physical instability may be triggered or suppressed numerically by changing the follow-up time of drift quantities relative to the period length or e-folding time of unstable oscillations. If drift quantities are changed slowly enough, the instabilities are smoothed away and we get reasonable stationary solutions [3]. On the contrary, if drift quantities are changed very fast, plasma instabilities are observed.

Three different types of instabilities have been recognized. We found that the strongest is the Kelvin–Helmholtz (KH) instability. If the KH instability is damped (by applying a special numerical procedure) then two types of interchange instabilities are observed. The behavior of the plasma instabilities is analyzed for a wide range of plasma parameters in the SOL and for different numbers of numerical grid spacing. According to the results obtained so far, the impurities have a negligible influence and do not produce significant

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changes of the instability growth rates and mode pattern.

2. Physical model and numerical method

The TECXY-code [3,4], like most other 2D computational plasma edge fluid models, is primarily based on the classical transport equations derived by Braginskij [5]. The transport along field lines is assumed to be classical [6,7], whereas radial transport is governed by drifts and anomalous transport processes with prescribed radial transport coefficients. The dynamics of deuterium and impurity neutrals in the SOL is described by an analytical model, which accounts for recycling of deuterons as well as for impurity sputtering at the limiter plates [3,4].

Equations for drifts and currents have been obtained from the radial and diamagnetic components of Ohm's law and the equation of motion in the form:

Perpendicular drift velocities:

$$v_\perp^a = -\frac{1}{Bh_y}\left[\frac{1}{eZ_a n_a}\frac{\partial p_a}{\partial y} + \frac{\partial \Phi}{\partial y}\right],$$
$$v_\perp^e = \frac{1}{Bh_y}\left[\frac{1}{en_e}\frac{\partial p_e}{\partial y} - \frac{\partial \Phi}{\partial y}\right]. \tag{1}$$

Radial drift velocities:

$$v_{yd}^a = \frac{h_\phi}{Bh_x}\left\{\frac{1}{eZ_a n_a}\frac{\partial p_a}{\partial x} + \frac{\partial \Phi}{\partial x} - \frac{m_a}{eZ_a}(v_\parallel^a)^2\frac{h_\theta}{h_x}\frac{\partial(h_x/h_\theta)}{\partial x}\right\} - \frac{m_a}{eZ_a n_a B}\left(S_{v_\perp}^a - m_a v_\perp^a S_n^a\right), \tag{2}$$

$$v_{yd}^e = -\frac{h_\phi}{Bh_x}\left(\frac{1}{en_e}\frac{\partial p_e}{\partial x} - \frac{\partial \Phi}{\partial x}\right). \tag{3}$$

Current components:

$$j_\perp = \sum_a eZ_a n_a v_\perp^a - en_e v_\perp^e,$$
$$j_y = \sum_a eZ_a n_a v_{yd}^a - en_e v_{yd}^e. \tag{4}$$

Here coordinates x and y correspond to the poloidal and radial directions, respectively; $a = i$ denotes deuterium ions and $a = j$ the different charge states of an impurity element ($j = 1, \ldots, 6$ in the case of carbon). In the present calculation we have neglected the influence of viscosity and curvature on drifts. In order to calculate the poloidal current j_x and parallel current $h_\theta j_\parallel \simeq j_x - j_\perp$ as well as the plasma potential, the condition $div = 0$ has been used together with the condition for a unique electrostatic potential Φ ($\oint \mathrm{d}\Phi = 0$) and Ohm's law.

We used standard boundary conditions in our calculations [3,5]. At the core boundary in most cases constant plasma parameters have been prescribed: plasma temperatures $60\ \mathrm{eV} \leqslant T_{\mathrm{ic}} \leqslant 240\ \mathrm{eV}$, $T_{\mathrm{ec}} = 0.5T_{\mathrm{ic}}$ and background ion density $5\times10^{18}\ \mathrm{m}^{-3} \leqslant n_{\mathrm{ic}} \leqslant 4\times10^{19}\ \mathrm{m}^{-3}$. In some cases, however, if an instability developed close to the core boundary, it has been necessary to use, instead of constant plasma parameters, constant input energy and particle fluxes as the boundary condition at the core contact. For impurity ions we have assumed the total carbon ion flux across the core boundary $\sum_{j=1}^{6}\Gamma_y^j = 0$ [3]. At the wall we have used decay lengths as boundary conditions for the densities and temperatures (with $\lambda = 2$ cm) [3]. Standard sheath boundary conditions for poloidal velocity and heat fluxes are used at the limiter plate [8,9]. The potential drops $\Delta\Phi$ across the Langmuir sheaths in front of both limiter sides are changed by the poloidal current according to: $e\Delta\Phi = e\Delta\Phi_0 - T_e \ln\left[1 - j_x/e\sum_a Z_a n_a v_x^a\right]$ where $e\Delta\Phi_0$ is the potential drop without a poloidal current [9]. At the auxiliary boundary in the transition layer we have assumed that all quantities are continuous and poloidally periodic.

The numerical method used in the code TECXY for solving fluid equations has been described in Ref. [4]. In order to find solutions to the plasma equations, we used a time relaxation method on a staggered mesh, which can be nonuniform in both directions, according to the simple prescription $h_i/h_{i+1} = w =$const, where h_i ($= \Delta x$ or Δy) is the mesh step. When trying to implement drifts into the TECXY code we encountered serious numerical difficulties, which were associated with a strong nonlinearity of the boundary conditions for the plasma potential and currents on both limiter sides, and with the fact that the equations for drifts and currents are steady state, whereas the fluid equations are time dependent. In order to overcome these numerical problems some kind of underrelaxation procedure for drift quantities was used in order to adjust slowly drifts and currents in time in the following way: $f(t+\Delta t) = \alpha f^s(t) + (1-\alpha)f(t)$ where f is some drift quantity (Φ, $v_\perp^a$, v_{yd}^a, Ψ,) and $f^s(t)$ is the stationary solution obtained with the plasma parameters at the moment t and $\alpha \equiv \mathrm{REL}\times\Delta t$ is the underrelaxation parameter. We found that in order to obtain the steady state solution, drifts should follow the time evolution of the plasma with some delay. If the relaxation parameter REL is large then plasma instabilities can develop, but for REL sufficiently large the characteristic time scale of the instability does not depend on the REL value.

3. Stationary solution

In order to analyze plasma instabilities, it was indispensable to find first steady state solutions for the plasma and impurity parameters in the TEXTOR tokamak boundary layer. This was done by choosing the REL parameter small enough (REL $\approx$ 5–7 $\times\ 10^3$). We have done calculations for a high density auxiliary heated TEXTOR discharge in deuterium with carbon as the dominant intrinsic impurity element. The belt limiter ALT-II is at $\theta = -45°$ position, the total magnetic field $B = 2.25$ T and the Shafranov shift $\Delta = 6$ cm. The anomalous transport in radial direction is determined by the coefficients $D_\perp^a(y) = 0.6(2 - (y/\Delta_T)^2)$ m^2/s, for $-\Delta_T < y < 0$, where $\Delta_T = 4$ cm is the transition layer width, and $\eta_\perp^a = \frac{1}{3}D_\perp^a$, $\chi_\perp^e/n_e = \frac{3}{2}\chi_\perp^a/n_a = 2D_\perp^a$ assuming a limit of $D_\perp^a = 1.2$ m^2/s in the SOL (for $0 < y < 4$ cm). For the present case the deuterium ion density at the core boundary ($y = -\Delta_T$) was $n_{\mathrm{ic}} = 10^{19}$ m^{-3}, the plasma ion temperature was $T_{\mathrm{ic}} = 240$ eV, and the recycling coefficient was $R = 0.8$.

In Fig. 1(a) we can clearly see large poloidal asymmetries in electron density between e-side and i-side of the limiter [3]. The asymmetries are due to drift flows and to the fact that poloidal drifts and anomalous radial diffusion are poloidally nonuniform according to the Shafranov shifted magnetic flux surfaces. The influence of the perpendicular drift velocity $v_\perp^i$ (within a magnetic surface) on the parallel flows can be seen in Fig. 1(b). Due to the drifts the parallel flow velocity is reduced for all ions on the e-side and enhanced on the i-side of the limiter, since $h_\theta v_\parallel^a \simeq v_x^a - v_\perp^a$.

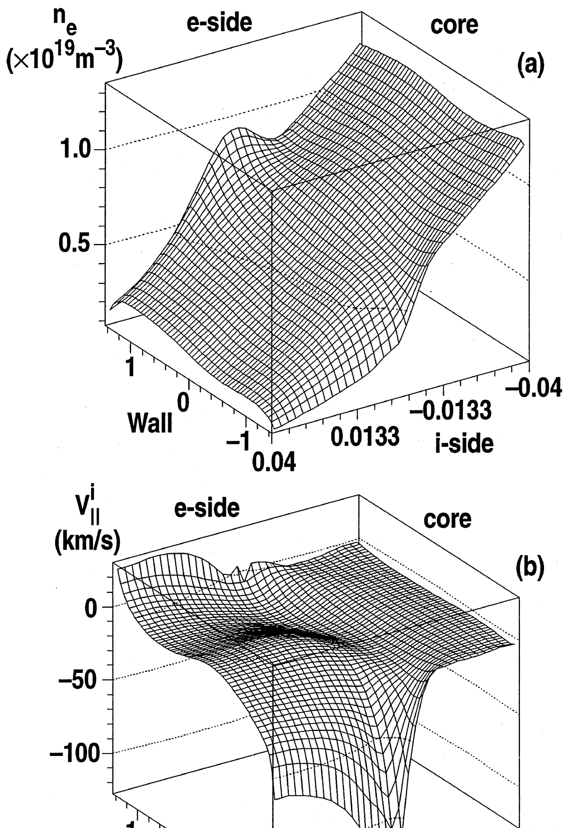

Fig. 1. Calculated profiles of plasma density n_e and parallel flow velocity $v_\parallel^i$ in the TEXTOR tokamak boundary layer

4. Kelvin–Helmholtz instability

As already explained, for sufficiently large REL parameter unstable plasma oscillations can be exited. It has been found that for the considered TEXTOR tokamak parameters at least three different types of instabilities can develop. It appears that the Kelvin–Helmholtz (KH) instability is the most dangerous one. It develops close to the i-side of the limiter, at the separatrix, where the values of the parallel velocity and its radial gradients are the largest. The time evolution of the diamagnetic drift velocity $v_\perp^i$ at the limiter ends is shown in Fig. 2 (for similar plasma parameters as in Fig. 1). We see that the typical growth time of this instability is of the order of ≈ 10 μs, as well as the oscillation period. We found that the development of instabilities is very similar in cases with and without impurities. Therefore, in the following we limit our considerations to the situation without impurities.

The Kelvin–Helmholtz instability in a magnetized plasma was analyzed theoretically by D'Angelo in 1965 [10]. The threshold for the onset of the KH instability can be found from the dispersion relation (see Ref. [10]) and written in the form

$$\kappa \equiv \frac{1}{\sqrt{2}c_s^i}\left|\frac{\partial v_\parallel}{\partial y}\right| L_n > 1, \tag{5}$$

where c_s^i is the ion sound speed and L_n is the e-folding length of the radial density profile. From the dispersion relation also the expressions for the oscillation period (τ_{os}) and the growth time (τ_i) can be deduced:

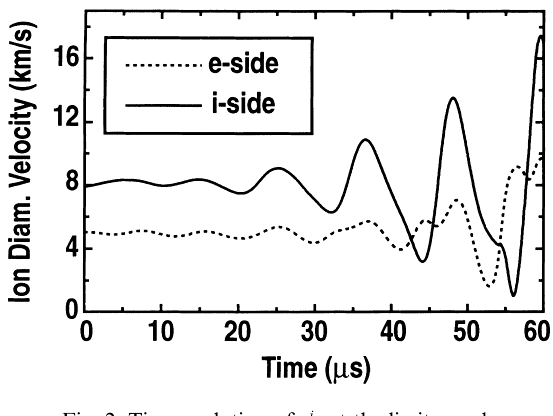

Fig. 2. Time evolution of $v_\perp^i$ at the limiter ends

$$\frac{1}{\tau_{os}} = \frac{1}{\lambda_x}\left(v_x - \frac{1}{enB}\frac{\partial p_i}{\partial y}\right) \approx \frac{1}{\lambda_x}\frac{2T_i}{eBL_n}, \quad (6)$$

$$\tau_i = \tau_{os}\frac{1}{\sqrt{2\pi^2(\kappa-1)}}. \quad (7)$$

For typical TEXTOR tokamak parameters (with $\lambda_x \approx$ 10 cm as allowed by the numerical grid) we obtain $\tau_{os} \approx$ 10 μs in good agreement with the numerical results.

In order to analyze the KH instability in more detail, we have performed numerical simulations for a wide range of input plasma parameters and different numbers of mesh steps. We found that the KH instability develops only, if the plasma temperature at the separatrix is sufficiently large since for plasma temperatures at the separatrix below $T_{is} \approx$ 80–70 eV, $T_{es} \approx$ 40–35 eV there is no instability. It should be stressed that we do not observe any dependence of the characteristic times (τ_{os}, τ_i) on the plasma density. We also point out that the threshold parameters κ [Eq. (5)] calculated for each case (Fig. 3) agree very well with the theoretical predictions, namely we found that κth $\approx$ 1–1.1, which means that for $\kappa > 1.1$ we have only unstable cases, whereas cases with $\kappa < 1$ are always stable. Concerning low plasma temperatures, we suppose that the stabilizing mechanism is associated with the increase of the radial drift velocity at the i-side with decreasing temperature (due to the decrease of the poloidal gradient lengths), which makes the radial density profile at the limiter end steeper, and according to the threshold formula [Eq. (5)] the KH instability is damped.

In Table 1 the dependence of the KH instability on the poloidal wave number is shown. Calculations have been performed assuming equidistant mesh in poloidal direction for different numbers of mesh points (41, 82, 123; the mesh in radial direction was nonuniform with $w_y = 1.1$) and for two different plasma densities at the core boundary. It can be seen that the oscillation period

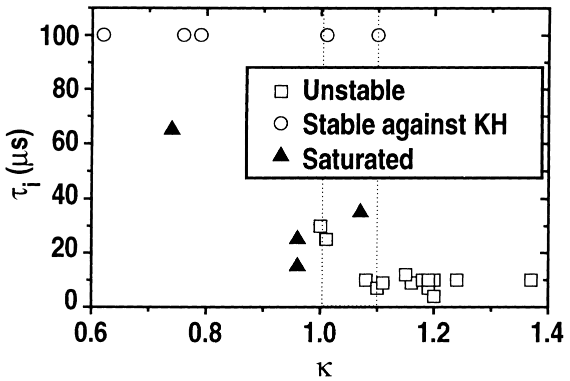

Fig. 3. KH instability growth times as a function of the threshold parameter κ [Eq. (5)] (for stable cases instead of $\tau_i = \infty$ we put $\tau_i = 100$ μs).

Table 1
Dependence of the KH instability on the poloidal wave number (in both cases $T_{ic} = 240$ eV)

No. of mesh points	$n_{ic} = 1 \times 10^{19}$ (m^{-3})	$n_{ic} = 2 \times 10^{19}$ (m^{-3})
41	Saturated	Stable
	$\kappa \cong 1.$	$\kappa \cong 1.1$
	$\tau_i \approx 70$ μs	
	$\tau_{os} \approx 20$ μs	
82	Unstable	Unstable
	$\kappa \cong 1.19$	$\kappa \cong 1.15$
	$\tau_i \approx 7$ μs	$\tau_i \approx 12$ μs
	$\tau_{os} \approx 14$ μs	$\tau_{os} \approx 12$ μs
123	Unstable	Unstable
	$\kappa \cong 1.2$	$\kappa \cong 1.1$
	$\tau_i \approx 4$ μs	$\tau_i \approx 7$ μs
	$\tau_{os} \approx 10$ μs	$\tau_{os} \approx 8.5$ μs

τ_{os} decreases almost inversely proportional to the wave number (increasing number of mesh points) in agreement with the theoretical predictions [Eq. (6)]. Similarly, also the growth time τ_i decreases, if the number of mesh points increases.

It is apparent from the threshold condition [Eq. (5)] that, if the e-folding length of the radial profile of the parallel velocity increases, then the KH instability is damped. In order to check, whether the observed instability is of the KH type, we have performed a numerical experiment. We flattened the radial profile of the parallel velocity by changing the mesh (from nonuniform to equidistant) and by doubling the e-folding length of the radial profile of the poloidal velocity v_x in the transition layer at the auxiliary boundary, which in our model is determined just by boundary condition. Fig. 4 shows a comparison between the standard case [Fig. 4(a)] and the case with longer radial gradient lengths [Fig. 4(b)] ($n_{ic} = 2 \times 10^{19}$ m^{-3}, $T_{ic} = 240$ eV). It is evident from the results that changes in the radial profile can damp the KH instability, since in the second case we have obtained a saturated state of the instability.

It is interesting that whenever we observe saturated states of the KH instability, we are usually very close to the stability threshold $\kappa \approx 1$. The saturated state is characterized by stable oscillations of plasma parameters. The observed fluctuation level is usually below 20% for the plasma density and very small for the temperatures, but it can be larger for the drift quantities. We suppose that the stabilizing mechanism is connected with the changes in the radial profile of the parallel velocity due to the oscillations of the diamagnetic drift velocity.

5. Interchange instability

The KH instability is so strong, that, if it develops, we cannot see any other type of instabilities. Since the

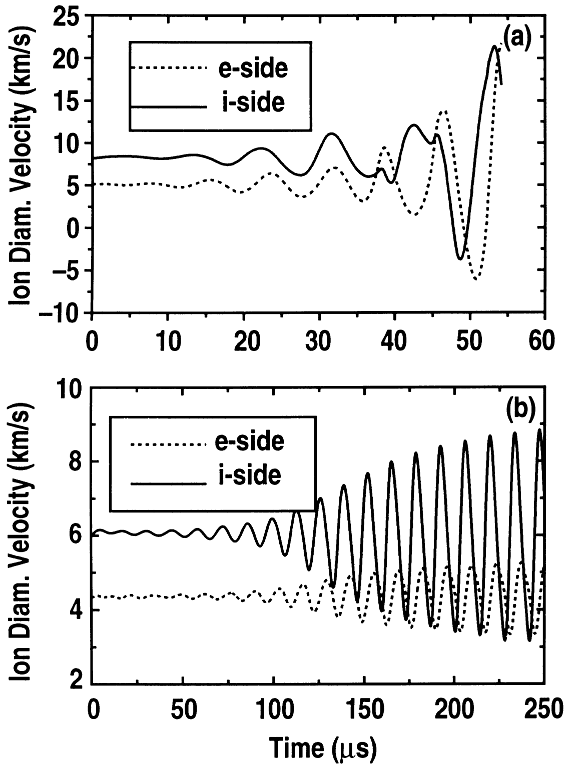

Fig. 4. Changing the radial profile of $v_{\|}^{i}$ can damp the KH instability: (a) standard mesh (steep profile of $v_{\|}^{i}$; $\kappa = 1.37$); (b) uniform mesh in radial direction (flat profile of $v_{\|}^{i}$; $\kappa = 0.96$).

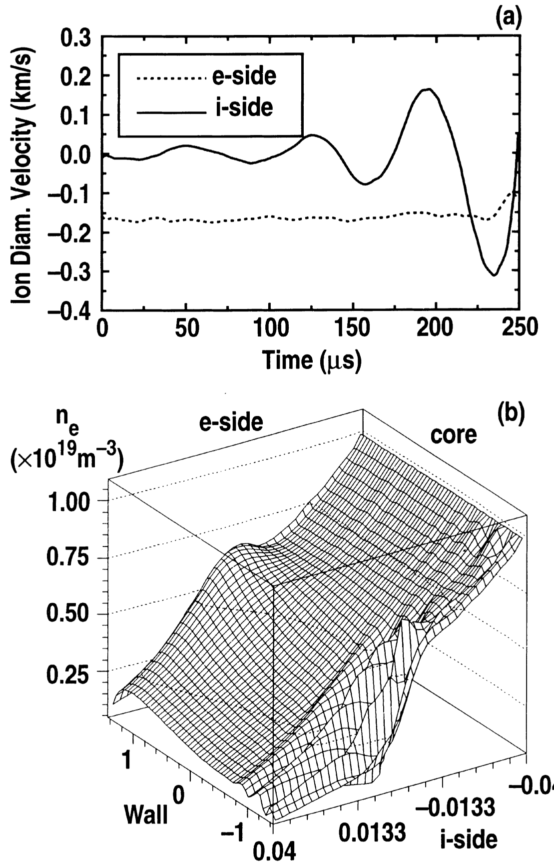

Fig. 5. (a) Time evolution of v_{y}^{i} at the limiter ends; (b) Profile of the plasma density at $t = 250$ ms (for this case $n_{ic} = 1 \times 10^{19}$ m^{-3}, $T_{ic} = 240$ eV)

KH instability develops in the radial direction, it was possible to kill it by applying a special damping of fluctuations of drift quantities in radial direction, similar to the radial double smoothing in Ref. [11].

After damping the KH instability we observed the interchange (IC) instability. The typical picture of this instability is displayed in Fig. 5. The characteristic time scale of the IC instability is much longer than for the KH instability ($\tau_i^{\mathrm{IC}} \approx 70$–170 μs) in agreement with a simple analytical estimate (Eq. (6) and Ref. [12]):

$$\gamma_i^{\mathrm{IC}} \simeq 2q\frac{T_e}{eB}\frac{\rho_i}{L_p}6\pi^2\lambda_x^2\frac{2T_i}{eB}\frac{\rho_i}{L_n};$$

$$\tau_i^{\mathrm{IC}} \approx \tau_{\mathrm{os}}^{\mathrm{KH}} \times \frac{\lambda_x}{60\rho_i} > 3\tau_{\mathrm{os}}^{\mathrm{KH}}, \quad (8)$$

where $q = 3$, $T_i = 2T_e$, $L_p \simeq L_n$ and ρ_i is the ion gyroradius. The IC instability preferably develops close to the i-side in the SOL, where for the considered plasma parameters the radial pressure gradient is very steep. There is also some short-wave fluctuation in the transition layer, which starts to develop near outboard midplane, where the magnetic field curvature is the worst, but in this specific case it is dominated by the instability developing at the i-side. We observed the interchange instability only for relatively high plasma temperatures ($T_{\mathrm{is}} > 110$ eV). It seems that the growth rates of the interchange instability follow the theoretical dependence on the wave number ($\gamma_i^{\mathrm{IC}} \propto k_x^{-2}$), but because of strong mixing between different kinds of IC modes (localized near i-side or outboard) it is difficult to give a definitive answer. Concerning the short-wave interchange oscillations it is clear that they can develop only, if the poloidal mesh step is sufficiently small, which makes the detailed analysis extremely difficult because of the necessary increase in computational time. Nevertheless, we found that they start to grow preferentially in regions of unfavorable curvature, close to the e-side in the transition layer, and then distribute uniformly in the transition layer (Fig. 6). If we carefully control the energy and particle fluxes, entering the boundary layer from the core, by choosing the time step sufficiently small, then the instability saturates and the oscillations are stable. However, in case we allow for some increase of the input particle and energy fluxes, then the instability suddenly develops, in the region close to outboard midplane. At

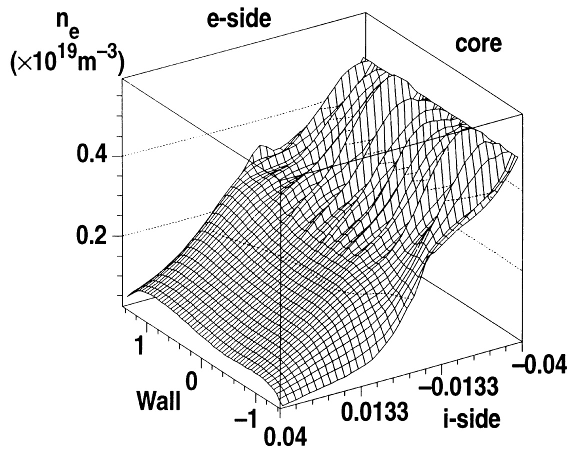

Fig. 6. Profile of n_e showing development of the plasma fluctuations in transition layer (for this case $n_{ic} = 5 \times 10^{18}$ m^{-3}, $T_{ic} = 240$ eV).

present we are not able to decide whether the sudden growth of fluctuations is numerical or physical.

6. Conclusions

The code TECXY [3] has been used to analyze the specific physical problems connected with the transport of plasma and impurities in the TEXTOR tokamak boundary layer in the presence of drifts and currents. It appeared that during the plasma evolution some unstable plasma oscillations can develop, which usually must be damped by a proper choice of the so called relaxation coefficient (REL) in order to obtain stationary solutions.

If, however, the REL coefficient is large enough, then plasma instabilities are allowed to develop. We have found that the most dangerous one is the Kelvin–Helmholtz instability, which arises at the ion side of the limiter. The typical growth time of the KH instability is of the order 5–15 μs, and the growth rate seems to change linearly with the poloidal wave number in agreement with theoretical predictions. The theoretical threshold for the KH instability has been confirmed by the numerical simulations. The KH instability stabilizes at low temperatures at the separatrix), probably due to the increase of the radial drift velocity at the ion limiter side, which leads to steeper density gradients.

If a special kind of radial damping of the fluctuations is introduced, then it is possible to kill the KH instability and consequently the interchange instability can show up. It arises preferentially in the SOL near the limiter i-side, where the radial pressure gradient is the largest. It has been found that the interchange instability appears only for relatively high plasma temperature ($T_{is} > 110$ eV).

We observed also short-wave oscillations (with 123 mesh points), which are localized in the transition layer. We found that if the boundary conditions at the interface with the core plasma are perfectly satisfied then the instability saturates, otherwise a very strong (maybe partly numerical) instability develops close to outboard midplane.

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